Acknowledgments

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Numerical Implementation of a Modified Liou-Steffen Upwind Scheme

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Introduction

SEVERAL recent efforts[1–3] have focused on the development of upwind schemes for the Euler equations that combine the accuracy of flux-difference splittings[4] in the capturing of shear layers with the robustness and low cost of flux-vector splittings.[5] Perhaps the most popular of these "hybrid" upwind algorithms is the advection upstream splitting method (AUSM) of Liou and Steffen.[2] In this approach, the inviscid flux at a cell interface is split into a convective contribution, which is upwinded in the direction of the flow, and a pressure contribution, which is upwinded based on acoustic considerations. The treatment of the convective portion of the flux allows the AUSM to capture a steady contact discontinuity without excess numerical diffusion. As a result, the AUSM can capture shear layers quite accurately, even with a first-order discretization. Under certain conditions, however, a first-order AUSM discretization will capture a strong normal shock in a nonmonotone fashion, effectively precluding the possibility of an accurate higher order extension in the shock region. Nevertheless, the AUSM would appear to be a viable alternative to more "standard" upwind approaches, both in terms of accuracy and efficiency.

As with most flux-vector splitting approaches, different forms for the AUSM interface flux are required for subsonic or supersonic values of the state Mach number. In addition, the evaluation of the convective portion of the interface flux for subsonic situations involves a discontinuous switching between left and right states. Potential users of the approach may be daunted somewhat by this seeming complexity, especially if an implicit formulation based on an exact linearization of the AUSM is needed. In this Note, a particular construction of the AUSM interface flux that does not require if-then-else Fortran logic to account for switches among the various states is presented. Based on this "compact" formulation, two linearizations, one nearly exact within a finite volume framework and the other an approximate, but much simpler, node-based formulation, are also presented. A simple method for "blending in" elements of the more dissipative Van Leer/Hänel flux-splitting scheme,[5] useful for eliminating the non-monotone behavior of the AUSM in the vicinity of a strong normal shock, is also included in the analysis. For reasons of simplicity and compactness, the development that follows considers only first-order interpolations from states i and $i+1$ on either side of the cell interface $i+1/2$ and assumes a two-dimensional flow of a single-component perfect gas. Unless otherwise noted, all quantities relating to metric derivatives, etc., are assumed to be evaluated at the cell interface.

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Interface Flux Definition

We begin by writing the inviscid flux in the kth generalized-coordinate direction ($k = \xi, \eta$) as the sum of convective and pressure components:

$$F \equiv F^c + F^p = \frac{|\nabla k|}{J} M \tilde{F}^c + \frac{|\nabla k|}{J} p \tilde{F}^p \tag{1}$$

where

$$\tilde{F}^c = \begin{bmatrix} \rho a \\ \rho u a \\ \rho v a \\ H a \end{bmatrix}, \qquad \tilde{F}^p = \begin{bmatrix} 0 \\ \tilde{k}_x \\ \tilde{k}_y \\ 0 \end{bmatrix} \tag{2}$$

$$\tilde{k}_{x,y} = \frac{k_{x,y}}{|\nabla k|} \tag{3}$$

$$H = \rho[\gamma e + (1/2)(u^2 + v^2)] \tag{4}$$

$$p = (\gamma - 1)\rho e \tag{5}$$

The contravariant Mach number M is given as

$$M = \frac{1}{a}(\tilde{k}_x u + \tilde{k}_y v) \tag{6}$$

where a is the speed of sound.

Following Ref. 2, we define an appropriate interface flux $F_{1/2}$ by considering the behavior of the convective and pressure components separately. The convective portion of the interface flux $F^c_{1/2}$ is given by

$$F^c_{1/2} = \frac{|\nabla k|}{J}(C^+ \tilde{F}^c_i + C^- \tilde{F}^c_{i+1}) \tag{7}$$

where

$$C^+ = \alpha^+_i(1.0 + \beta_i)M_i - \gamma^+_{1/2}(1.0 - \delta_{1/2}) \times (\beta_i M^+_i + \beta_{i+1} M^-_{i+1}) - \delta_{1/2}\beta_i M^+_i \tag{8}$$

and

$$C^- = \alpha^-_{i+1}(1.0 + \beta_{i+1})M_{i+1} - \gamma^-_{1/2}(1.0 - \delta_{1/2}) \times (\beta_i M^+_i + \beta_{i+1} M^-_{i+1}) - \delta_{1/2}\beta_{i+1} M^-_{i+1} \tag{9}$$

In the preceding equations, the split Mach number $M^{\pm}$ is defined as

$$M^{\pm}_{i,i+1} = \pm(1/4)(M^{\pm}_{i,i+1} \pm 1.0)^2 \tag{10}$$

The functions $\alpha^{\pm}$ and β provide the correct sonic-point transition behavior:

$$\alpha^{\pm}_{i,i+1} = (1/2)[1.0 \pm \mathrm{sgn}(M_{i,i+1})] \tag{11}$$

$$\beta_{i,i+1} = -\max[0.0, 1.0 - \mathrm{int}(|M_{i,i+1}|)] \tag{12}$$

whereas the function $\gamma^{\pm}_{1/2}$ accounts for the dependence of the AUSM subsonic convective flux on the direction of the fluid motion:

$$\gamma^{\pm}_{1/2} = (1/2)[1.0 \pm \mathrm{sgn}(M^{+}_{i} + M^{-}_{i+1})] \tag{13}$$

In the preceding expressions, sgn is the transfer-of-sign function and int is the Fortran integer truncation function.

The parameter $\delta_{1/2}$ provides a means of switching between Van Leer/Hänel and AUSM representations of the subsonic convective interface flux. For $\delta_{1/2} = 0.0$, the AUSM variant is obtained, whereas for $\delta_{1/2} = 1.0$, the Van Leer/Hänel form results. A variable form of $\delta_{1/2}$, dependent on an appropriate definition of an interface Mach number $M_{1/2}$, can be used to "blend" the AUSM and Van Leer/Hänel formulations, thus providing a means of eliminating oscillations in the vicinity of a strong normal shock while preserving the ability of the AUSM to recognize a steady contact discontinuity. To insure that the added dissipation only affects the sonic-point transition, a particular construction of $\delta_{1/2}(M_{1/2})$ should satisfy the following conditions: $M_{1/2} = M_{1/2}(|M_i|, |M_{i+1}|)$, $\delta_{1/2}(0) = 0.0$, and $\delta_{1/2}(1) = 1.0$. One might also require that $\partial\delta_{1/2}/\partial M_{1/2} \geq 0$ over the interval [0, 1], with the equality holding at $M_{1/2} = 0$ and at $M_{1/2} = 1$ to insure that smooth behavior occurs at the critical sonic and stagnation points. If, in addition to these criteria, one wishes to specify the value of $\delta_{1/2}$ corresponding to a selected value of $M_{1/2}$ within the interval, one must consider functional forms for $\delta_{1/2}$ that are not simple polynomials. A particular form for $\delta_{1/2}(M_{1/2})$, based on the conditions outlined earlier and suitable for reacting-gas re-entry computations, is given as follows:

$$M_{1/2} = \min[1.0, |M_i|, |M_{i+1}|] \tag{14}$$

$$\delta_1 = \sqrt{M_{1/2}(2.0 - M_{1/2})} \tag{15}$$

$$\delta_2 = 64.0 M^3_{1/2} \tag{16}$$

$$\delta_{1/2} = \delta_1 \tanh\left[\frac{\delta_2}{\delta_1 + \varepsilon}\right] \tag{17}$$

where ε is a small number designed to prevent division by zero.

The pressure contribution to the interface flux is defined using the characteristic-based splitting of Ref. 2:

$$F^{p}_{1/2} = \left|\frac{\nabla k}{J}\right| \tilde{F}^{p}(D^{+}_{i} p_i + D^{-}_{i+1} p_{i+1}) \tag{18}$$

where

$$D^{\pm}_{i,i+1} = \alpha^{\pm}_{i,i+1}(1.0 + \beta_{i,i+1}) - \frac{\beta_{i,i+1}}{2}(1.0 \pm M_{i,i+1}) \tag{19}$$

Interface Flux Linearization

Implicit techniques for integrating the unsteady Navier-Stokes equations can generally be viewed as approximations to Newton's method for solving the steady Navier-Stokes system. A local time linearization of the nonlinear system is usually required, as is a matrix inversion procedure (either exact or approximate) for solving the linear system at each time level. The rate of convergence of the integration procedure is strongly affected by the accuracy of both the linearization and the matrix factorization.

We can construct a first-order linearization of the interface flux $F_{1/2}$ in terms of the variable set $V = [v_1, v_2, v_3, v_4]^T$ by defining

$$\frac{\partial F_{1/2}}{\partial V_{1/2}} \Delta V_{1/2} = (A^{+}_{c} + A^{+}_{p})_{1/2} \Delta V_i + (A^{-}_{c} + A^{-}_{p})_{1/2} \Delta V_{i+1} \tag{20}$$

where

$$(A^{+}_{c})_{1/2} \equiv \frac{\partial F^{c}_{1/2}}{\partial V_i}, \qquad (A^{-}_{c})_{1/2} \equiv \frac{\partial F^{c}_{1/2}}{\partial V_{i+1}}$$
$$(A^{+}_{p})_{1/2} \equiv \frac{\partial F^{p}_{1/2}}{\partial V_i}, \qquad (A^{-}_{p})_{1/2} \equiv \frac{\partial F^{p}_{1/2}}{\partial V_{i+1}} \tag{21}$$

For the convective interface flux, a term-by-term differentiation of Eq. (7) (holding α, β, γ, and δ locally constant) gives the following:

$$(A^{+}_{c})_{1/2} \equiv \frac{|\nabla k|}{J}\left[C^{+}\frac{\partial \tilde{F}^{c}_{i}}{\partial V_i} + \left(\tilde{F}^{c}_{i}\frac{\partial C^{+}}{\partial M_i} + \tilde{F}^{c}_{i+1}\frac{\partial C^{-}}{\partial M_i}\right)x^{T}_{i}\right] \tag{22}$$

$$(A^{-}_{c})_{1/2} \equiv \frac{|\nabla k|}{J}\left[C^{-}\frac{\partial \tilde{F}^{c}_{i+1}}{\partial V_{i+1}} + \left(\tilde{F}^{c}_{i}\frac{\partial C^{+}}{\partial M_{i+1}} + \tilde{F}^{c}_{i+1}\frac{\partial C^{-}}{\partial M_{i+1}}\right)x^{T}_{i+1}\right] \tag{23}$$

where

$$x^{T}_{i,i+1} = \left[\frac{\partial M}{\partial v1}, \ldots, \frac{\partial M}{\partial v4}\right]_{i,i+1} \tag{24}$$

The Jacobian matrices $\partial\tilde{F}^{c}_{i,i+1}/\partial V_{i,i+1}$ can be computed in a straightforward fashion, as can the scalar derivatives $\partial C^{\pm}/\partial M_{i,i+1}$. For the latter, we have

$$\frac{\partial C^{+}}{\partial M_i} = \alpha^{+}_{i}(1.0 + \beta_i) - \frac{\beta_i}{2}(1.0 + M_i)[\gamma^{+}_{1/2}(1.0 - \delta_{1/2}) + \delta_{1/2}] \tag{25}$$

$$\frac{\partial C^{-}}{\partial M_i} = -\frac{\beta_i \gamma^{-}_{1/2}}{2}(1.0 + M_i)[1.0 - \delta_{1/2}] \tag{26}$$

$$\frac{\partial C^{+}}{\partial M_{i+1}} = -\frac{\beta_{i+1}\gamma^{+}_{1/2}}{2}(1.0 - M_{i+1})[1.0 - \delta_{1/2}] \tag{27}$$

$$\frac{\partial C^{-}}{\partial M_{i+1}} = \alpha^{-}_{i+1}(1.0 + \beta_{i+1}) - \frac{\beta_{i+1}}{2}(1.0 - M_{i+1})[\gamma^{-}_{1/2}(1.0 - \delta_{1/2}) + \delta_{1/2}] \tag{28}$$

An alternative, node-based linearization of the interface flux can be written as

$$\frac{\partial F_{1/2}}{\partial V_{1/2}} \Delta V_{1/2} = (A^{+}_{c} + A^{+}_{p})_{i} \Delta V_i + (A^{-}_{c} + A^{-}_{p})_{i+1} \Delta V_{i+1} \tag{29}$$

In contrast to the preceding development, the "positive" and "negative" flux Jacobians for the node-based linearization are allowed to depend only on fluid and geometric properties at states i and $i + 1$, respectively. Appropriate "nodal" forms for the AUSM convective flux Jacobians can be obtained by contracting the expressions in Eqs. (7–28) to a single state i. The result is a less exact but much simpler approximation:

$$(A^{\pm}_{c})_{i} = \frac{|\nabla k|}{J}\left\{[\alpha^{\pm} M(1.0 + \delta\beta) - \delta\beta M^{\pm}]\frac{\partial \tilde{F}_c}{\partial V} + D^{\pm}\tilde{F}_c x^{T}\right\}_i \tag{30}$$

Expressions for the linearization of the pressure flux $F^{p}_{1/2}$ [$A^{\pm}_{p}$ in Eqs. (20) and (29)] can be obtained by a process similar to that outlined earlier. For the node-based procedure, we have

$$(A^{\pm}_{p})_{i} = \frac{|\nabla k|}{J}\tilde{F}^{p}\left[D^{\pm} y^{T} \mp \frac{p\beta}{2} x^{T}\right]_i \tag{31}$$

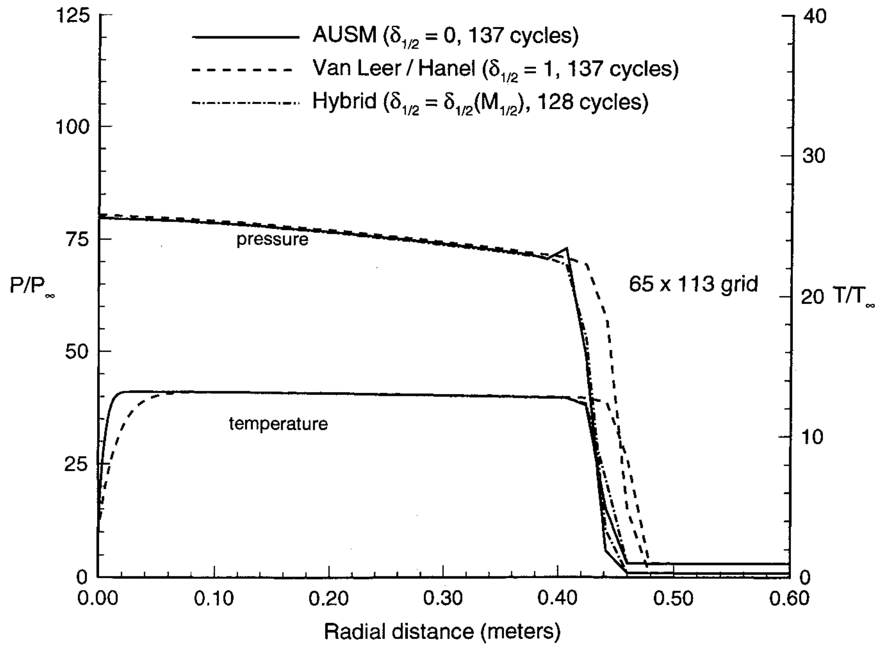

Fig. 1 Stagnation-line pressure and temperature distributions (65 × 113 grid; 2.5 km/s and 60-km altitude).

where

$$y^T = \left[\frac{\partial p}{\partial v1}, \ldots, \frac{\partial p}{\partial v4}\right] \tag{32}$$

Results

The ideas discussed in the preceding sections have been incorporated into an implicit computational fluid dynamics code suitable for computing two-dimensional/three-dimensional perfect gas viscous flows. A finite volume discretization of the Navier-Stokes set is employed, with the inviscid fluxes treated as outlined in Eqs. (7) and (18). The algorithm utilizes the simpler, node-based linearization of Eqs. (30) and (31) (written in terms of the primitive-variable vector $[\rho, u, v, w, e]^T$) to define a line/planar Gauss-Seidel smoother for a multigrid integration strategy. Figure 1 shows stagnation-line pressure and temperature distributions corresponding to viscous flow over a two-dimensional cylinder (2.5 km/s and 60-km altitude). For $\delta_{1/2} = 0$, corresponding to the basic first-order AUSM approach, a thin boundary layer is predicted, but the pressure distribution is nonmonotone. The Van Leer/Hänel approach ($\delta_{1/2}$ = 1.0) captures the shock monotonically but is too dissipative in the viscous layer, leading to an incorrect prediction of the shock-standoff distance. The "blended" formulation defined by Eqs. (14–17) captures the shock smoothly without thickening the viscous layer. Also shown in Fig. 1 is the number of multigrid cycles required for convergence (a normalized residual reduction of five orders of magnitude). For the blended approach, there is no evidence of a relative degradation in convergence rate caused by the omission of the derivatives of $\delta_{1/2}$ in the implicit formulation.

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Fuzzy Controller and Neuron Models of Harten's Second-Order Total Variation Diminishing Scheme

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Introduction

OVER the last several years, the use of fuzzy controllers and neural networks in industrial and consumer products has been increasing. These uses include reading handwritten addresses, analyzing satellite data, and stabilizing hand-held video camera images. It is proposed here to expand the use of fuzzy controllers and neural networks into the field of computational fluid dynamics (CFD). In this Note, Harten's second-order total variation diminishing (TVD) scheme is described as a fuzzy controller and as a single-input single-output neuron.

Numerical algorithms in CFD involve complicated discretizations of the governing partial differential equations such as the Euler equations of fluid mechanics. The Euler equations are hyperbolic and solution techniques generally involve knowledge of the wavelike behavior of the equations. Sophisticated algorithms incorporate flux limiters which control the amount of flux each wave produces to give the desired result. In general, flux limiters are required to ensure that the solution is physically realizable (e.g., does not violate the second law of thermodynamics). Sweby[1] describes flux limiters in a figure which graphically depicts regions where algorithms are TVD and where they are second-order-accurate TVD. Several well-known flux limiters are shown in this context and the limiter used by Harten,[2] the minmod function, is shown to be a boundary of the second-order-accurate TVD region. All of the limiters are monotone, nondecreasing functions.

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